A NUMERICAL METHOD TO SYNTHESIZE THE ELEMENT CHARACTERISTIC IN ANALOG CIRCUIT DESIGN

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ABSTRACT
This paper introduces a numerical method with which it is possible to synthesize the element characteristic of a not specified circuit element or even a circuit block (“black box”) in a way that certain circuit specifications are fulfilled. The element characteristic is computed in a given interval of a variable which follows from the chosen analysis method (e.g. DC sweep, temperature analysis). This allows for realizing the black box such that circuit specifications which depend on the corresponding variable are valid over a variable range of interest. The paper explains the numerical method and afterwards shows two applications of the method for achieving the temperature independency and a given DC-input-output behavior of a circuit, respectively. Computations are carried out with Analog Insydes’ DAE\(^1\) solver [1]. Analog Insydes (“INtelligent SYmbolic Design System”) [2] has been developed as an Add-on to the computer algebra system Mathematica [3] and is a toolbox for modeling, analysis and design of analog electronic circuits.

1. THE NUMERICAL METHOD
Following a method will be described that directly works with symbolic network equations which are automatically generated from a hierarchical netlist format applying Analog Insydes. This property allows for a flexible handling of mathematical equations which is not possible with traditional simulators (e.g. PSpice).

The development of the method can be subdivided into four major steps:

I) Starting point of the method is a circuit with a fully designed topology except for the circuit element or block \( \mu \) of interest. The element \( \mu \) shall be described by the functional relation \( G_\mu: [\lambda_1, \lambda_2] \to \mathbb{R} \) with \( \lambda_1, \lambda_2 \in \mathbb{R} \). This yields the following element relation \( f_\mu \) for the black box where this paper distinguishes between three different cases, \( G_\mu \) can occur:

\[
\begin{align*}
\text{a)} & \quad f_\mu(i_\mu, v_\mu, x) = F_\mu(i_\mu, v_\mu, x, G_\mu(i_\mu)) \\
\text{b)} & \quad f_\mu(i_\mu, v_\mu, x) = F_\mu(i_\mu, v_\mu, x, G_\mu(v_\mu)) \\
\text{c)} & \quad f_\mu(i_\mu, v_\mu, x) = F_\mu(i_\mu, v_\mu, x, G_\mu(x))
\end{align*}
\]

\( x \) is the integration variable, \( i_\mu \), and \( v_\mu \) are the branch currents, and the branch or node voltages of the element, respectively. In case of a transient analysis the integration variable \( x \) corresponds to the time \( t \), in case of a temperature analysis to the temperature \( T \). For parametric analyses \( x \) matches the varied parameter, e.g. the value of an independent voltage or current source (DC sweep).

II) The second step transforms the wanted characteristic \( G_\mu \) into the function \( g_\mu(x) \) as \( G_\mu \) is not necessarily a function of the integration variable \( x \). This step is required as the DAE solver used for the numerical computations assumes all system variables to be a function of the integration variable. Thus, the following transformations have to be carried out:

\[
\begin{align*}
\text{a)} & \quad G_\mu(i_\mu(x)) = G_\mu(H_\mu(x)) = g_\mu(x) \\
\text{b)} & \quad G_\mu(v_\mu(x)) = G_\mu(H_\mu(x)) = g_\mu(x) \\
\text{c)} & \quad G_\mu(x) = g_\mu(x)
\end{align*}
\]

Additionally it has to be taken into account that for the cases a and b the functional relation \( H_\mu(x) \) is considered in the system of network equations. Otherwise a back-transformation of \( g_\mu(x) \) into \( G_\mu \) is not possible. More detailed information can be found in Section 3.

III) It is the aim of the method to numerically compute the characteristic of the element \( \mu \). For that pur-
pose \( g_\mu \) is interpreted as a variable. With this additional variable it follows that the degree of freedom of the circuit equations is increased. Obviously, the system of equations is now undetermined. To obtain unique solutions for the circuit variables, it is thus necessary to add a corresponding number of constraints \( Z_\mu \) to the system of circuit equations in order to bind the degrees of freedom. The constraints are appropriately chosen as specifications of the circuit, where the specifications have to be a function of the integration variable \( x \). Now, appending these additional constraints to the system of circuit equations yields the following extended set of equations:

\[
M[i, v, g_\mu, x] = 0 \\
Z_\mu[i, v, x] = 0
\]  

(3)

where \( M \) is the vector of nonlinear, differential-algebraic circuit equations formulated as function of the branch currents \( i \), the branch or node voltages \( v \), as well as \( g_\mu \) and \( x \).

IV) In the last step of the method the extended DAE system is solved for \( x \). For the cases a and b \( G_\mu \) is computed from the solutions obtained for \( g_\mu(x) \) and \( H_\mu(x) \) with the restriction that \( H_\mu'(x) \) must exist.

\[
a) \quad i_\mu = H_\mu(x), \quad v_\mu = g_\mu(x) \quad \Rightarrow \quad G_\mu(i_\mu) \\
b) \quad v_\mu = H_\mu'(x), \quad i_\mu = g_\mu(x) \quad \Rightarrow \quad G_\mu(v_\mu) \\
c) \quad g_\mu(x) = G_\mu(x)
\]

(4)

The synthesized functional relation is equivalent to the element characteristic of the circuit element or block of interest.

2. EXAMPLE A: BAND-GAP CIRCUIT

The above described numerical method is now demonstrated in the two following sections. Let’s start with an example where the wanted characteristic \( G_\mu \) is a function of the integration variable \( x \) (cf. case c in Section 1), so we will not have to make any transformations.

Figure 1 shows a band-gap reference circuit consisting of four bipolar transistors. The main goal is to model the resistor \( R_2 \) such that the output voltage measured at node OUT is independent of the temperature \( T \). Following, the temperature dependency of \( R_2 \) is modeled by a second order approximation:

\[
R_1(T) = R_0 \{1 + TC_{11}(T - T_0) + TC_{21}(T - T_0)^2\} \quad \text{(5)}
\]

where \( T_0 \) is the reference temperature, \( R_0, TC_{11}, \) and \( TC_{21} \) are constants, respectively. The bipolar transistors are modeled with the Gummel-Poon transistor model [4].

The circuit or more precisely the resistor \( R_2 \) shall now be sized with the constraint of a constant output voltage in a given temperature range. Therefore, we apply the above described numerical method. For the example \( G_\mu(x) \) as well as \( g_\mu(x) \) are equivalent to \( R_2(T) \). Instead of the black box a resistor with the symbolic value \( R_2 \) is introduced which involves a degree of freedom in the system of network equations due to the additional variable \( g_\mu \). The temperature independency of the output voltage \( V_{OUT} \) can be formulated as the constraint \( Z_\mu \):

\[
V_{OUT}'(T) = 0.
\]

(6)

This equation is added to the set of symbolic circuit equations in order to bind the degree of freedom. Next, the resulting DAE system of 46 equations and 46 variables is solved numerically within the temperature range \( T \in [300 \text{ K}, 400 \text{ K}] \) applying Analog Insydes’ DAE solver [1]. Figure 2 shows a plot of the given tem-
temperature dependency of the resistor \( R_1 \) (dashed line) and the synthesized temperature dependency of the resistor \( R_2 \) (solid line) within the investigated interval. The synthesized element characteristic for \( R_2 \) describes the exact temperature dependency a circuit element must have in order to fulfill the given constraint (6). A cross-check simulation with the synthesized characteristic in fact shows that the output voltage is temperature independent within the investigated interval (see Figure 3).

Now that we know the ideal element characteristic of the resistor, the next step of the design process would be to find a realization of the temperature-dependent resistor which still fulfills the given constraint. Therefore, the synthesized element characteristic for \( R_2 \) is approximated, for example, by a second order polynomial like given for \( R_1 \) in equation (5). With the values found for the constants \( R_0 \), \( TC_1 \), and \( TC_2 \) it is now possible to verify the achieved results applying a common circuit simulator. A cross-check simulation with PSpice yields a deviation of \( \Delta V_{\text{OUT}} = 0.3 \text{ mV} \) for the output voltage within the temperature interval \( T \in [300 \text{ K}, 400 \text{ K}] \) (see Figure 4). Finally the task remains to realize the resistor with the values found for above constants.

3. EXAMPLE B: OUTPUT STAGE

This section shows another application of the numerical method which is demonstrated for the example of an output stage consisting of three bipolar transistors (see Figure 5). Here, the wanted characteristic \( G_\mu \) is not a function of the integration variable \( x \) in contrast to the example in Section 2.

The task of the circuit is to couple the transistors \( Q_2 \) and \( Q_3 \) such that the output voltage \( V_{\text{OUT}} \) is proportional to the input voltage \( V_{\text{IN}} \). The element characteristic of the circuit block marked as a black box in the schematics, here a I-V-relation, shall be synthesized with the following specification:

\[
V_{\text{OUT}}'(V_{\text{IN}}) = \text{const} \\
V_{\text{OUT}}(0) = 0
\]  

The above described method is now carried out for a DC sweep of the input voltage in the range \( V_{\text{IN}} \in [-4.5 \text{ V}, 4.5 \text{ V}] \). In this example the wanted functional relation \( G_\mu \) is not a function of the integration variable \( x \), or to be more specific: the I-V-relation of the black box is not a function of \( V_{\text{IN}} \). Therefore, we introduce an independent voltage source instead of the black box with the unknown voltage \( V_{\text{O}}(V_{\text{IN}}) \). The corresponding branch current \( I_{\text{O}}(V_{\text{IN}}) \) is automatically added to the vector of circuit variables when setting up circuit equations (this is valid for modified nodal as well as sparse-tableau formulation). With this impedance transformation \( g_\mu(x) \) is equivalent to \( V_{\text{O}}(V_{\text{IN}}) \) and \( H_\mu(x) \) is equivalent to \( I_{\text{O}}(V_{\text{IN}}) \), respectively (cf. case a in Section 1). Alternatively, an admittance transformation could have been made, where we would replace the black box by an independent current source with the value \( IO(V_{\text{IN}}) \). Additionally, it has to be taken into account that the corresponding branch voltage

![Figure 3: Simulation with Analog Insydes showing the temperature-independent output voltage](image1)

![Figure 4: Simulation with PSpice showing the temperature-independent output voltage](image2)

![Figure 5: Schematics of the output stage](image3)
$VO(VIN)$ is considered in the system of network equations (cf. case b in Section 1).

The formulation of the constraint given in (7) matches a boundary value problem. In order to solve the system of network equations applying Analog Insydes’ DAE solver the problem has to be converted into an initial value problem. Therefore, we start with computing the DC solution of the system. This yields combined with the boundary condition in (7) the constant gradient of the voltage gain. Afterwards the list of symbolic circuit equations is extended by the constraint $Z_y$ and the resulting DAE system of 41 equations and 41 variables (transistors are modeled with the Gummel-Poon transistor model) is solved numerically for a DC sweep of the input voltage within the range $VIN \in [-4.5 \text{ V}, 4.5 \text{ V}]$. We obtain solutions for $VO(VIN)$ and $IO(VIN)$, which we can now transform into the wanted element characteristic $VO(IO)$. Figure 6 illustrates the synthesized I-V-relation of the black box which a circuit element must have in order to fulfill the demanded constraint.

A cross-check simulation using the synthesized characteristic confirms the linear DC transfer characteristic within the investigated parameter sweep (see Figure 7). Literature (e.g. [5]) frequently proposes an independent voltage source as realization for the black box, which can be acknowledged by the result illustrated in Figure 6 and by a cross-check simulation with PSpice.

Figure 8 shows the DC transfer characteristic for a swept independent voltage source $VO$ within the parameter range $VO \in [1.0 \text{ V}, 2.0 \text{ V}]$. The PSpice simulation yields an ideal value of 1.4 V for the independent voltage source.

### 4. CONCLUSIONS

This paper presented a new numerical method which allows for synthesizing the element characteristic of a not specified circuit element such that certain circuit specifications are fulfilled. It has to be pointed out that the method is no optimization, but an exact numerical computation. It was shown that the method can be helpful in order to design circuit elements by simply considering their ideally synthesized element characteristic.

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### 6. REFERENCES