

# The Practical Approach to Maths Education

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As makers of the well-known and widely used *Mathematica* maths computation system, we are involved with mathematics from most imaginable perspectives: as researchers of mathematics and employers of mathematicians, as well as in training, selling, and marketing to technical people across industry, government, and education sectors. Our opinions are therefore based on the breadth of this experience.

Over the last 15 years, we have watched the extraordinary range of uses to which *Mathematica* and therefore maths are put. Many are expected: physical sciences, engineering, finance, economics, and of course maths itself. Others are more surprising: biological and social sciences, management, linguistics, and art to name a few. Perhaps more importantly, the complexity of mathematical computations and concepts utilised is increasing in almost every field.

This demonstrates the tremendous and growing importance of mathematics as a practical necessity to our society. It also illustrates the vast number and variety of professionals other than mathematicians who use mathematics in their work. *Mathematica* sales should be fairly representative, being only about 8% to those who class themselves as mathematicians (pure or applied).

As the vast majority of students will use their mathematics for practical subjects, we therefore feel strongly that mathematics should be taught at school level primarily as a practical subject to prepare students for likely uses, not as an abstract subject or purely intellectual pursuit.

Paradoxically, by following this practical approach, many students find the subject more intellectually stimulating—an observation frequently made by those running innovative courses with *Mathematica*.

And for the small numbers who wish to study higher mathematics for its own sake, even the most theoretical of school courses today will not avoid the later need to relearn underlying concepts from first principles. It's hard to see how it could be otherwise: for example, the treatment of numbers for a 10 year old would certainly need relearning during a number theory course at university.

If one accepts that maths education is for practical purposes, there are major consequences for what should be taught and how. To a large extent people think of maths as computation—sometimes even just numerical calculations or arithmetic. Yet that is rarely where the use of maths fails in practical, real-world situations.

We would define maths more broadly as the process of (1) translating a problem to a mathematical form; (2) deciding what result is required mathematically; (3) doing the computation (i.e. moving from the mathematical beginning point to the mathematical end point); and (4) interpreting the result.

Most of maths education is spent on step 3. Yet this is the only step that can effectively be done by computer; indeed nowadays almost any computation (from symbolic algebra and calculus to matrices, statistics, and geometry) is done better by computer than by any student even after years of training. It is also the step that few professionals in any field utilising maths perform manually.

Instead it is the lack of skills in the other steps that critically fails industry—and also higher education in cases where computerised computation systems are utilised. Indeed industry sees it as a benefit or even a necessity for recruits to know how to use computers for doing maths. This in itself is an important part of a student's education.

The suggestion is not that no manual computation should be taught, but that the focus should be on its use as a means to an end, not an end in itself, and that computing technology should take over for that step after basic concepts of the computation are understood.

Traditionalists sometimes argue that so doing would reduce the intellectual content of maths education and turn it into some kind of brainless “button-pushing”. They are right to be concerned, but no more than they should be about non-computerised maths teaching resulting in the mindless application of manual computation processes—processes that are unlikely to be used outside education.

A key factor in ensuring that computers improve conceptual understanding is to use them in the most open-ended way possible. The more scope the student has for deciding how to employ the computer as a tool for solving a problem, the more the student will learn how to conceptualise and clarify it. Typical calculators have a fixed scope and so offer only a partially open-ended approach. Multimedia software designed to make less work for the teacher is more closed-ended through channelling students more narrowly, and is the most likely to cause problems that the traditionalists fear.

We believe in as open-ended an approach as possible, although we accept that there can be practical restrictions in applying it.

We have experience with the full spectrum of approaches from around the world. *Mathematica* used in its natural state is just about the most open-ended system a student could encounter. However it is also used to make environments that optimise the function of a more closed-ended style, for example guiding students through solutions, computing problems for them to answer, and assessing their capabilities. This latter approach requires less teacher time (a good example being Open University's Physica). Studies suggest that the open-ended approach substantially benefits conceptual understanding but requires just as much teacher time as current manual methods.

There seem to be two main reasons for the offered improvements in conceptual understanding. Firstly, to get a computer maths system to do a computation, one has to specify the problem precisely and be clear what computations are required to get the desired result. Doing maths by hand, it is much easier to blur these boundaries between the steps outlined above and “muddle through”. Computations are entered into before the problem is fully specified; often the wrong answer is obtained with little necessity to reset the concept. Secondly, once a type of calculation has been established a computer allows different numbers or expressions to be tried with minimal additional effort, establishing patterns of behaviour as the parameters change.

Many complicated, real-world examples can be computed this way and the student gains vastly more experience of how concepts play out in practice. Most importantly, computer maths provides for teaching from an experimental point of view, with students discovering relationships and concepts for themselves not by learning from first principles first. (Sarah Flannery’s book *In Code: A Mathematical Journey* describes how she became Irish Young Scientist of the Year with this approach.)

Another important consequence about which we hear is the ability to reorder the curriculum by conceptual difficulty of the subject matter rather than its computational complexity. For example, *Mathematica*-based classes often teach the application of calculus sooner because they can avoid the difficulty in calculating that usually delays this topic.

Perhaps the most exciting outcome we have observed from the *Mathematica* courses that take the approach advocated here is one of the most sought after: students find their maths more fun!

I hope this outline makes a strong case for a rethink of school mathematics based on full integration of computer maths tools. There is no suggestion that this is an easy change, at the very least requiring a rethink of syllabuses, examinations, teaching methodology, and coursework. But successfully implemented, the consequences would be far-reaching for students, industry, and research.

In summary, we could expect:

- A greater proportion of students taking maths
- More conceptual understanding of the topics covered or a broader range of topics being covered by a given point
- Practical topics covered earlier because they can be ordered by conceptual difficulty rather than computational complexity (e.g. many aspects of calculus)
- Greater experience and ability at simulating the real world
- More effective and extensive use of mathematics in the workplace